# RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

**B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2019**

**THIRD YEAR [BATCH 2017-20]**

Date : 19/12/2019 **MATHEMATICS [Honours]**

Time : 11 am – 3 pm **Paper : VI** Full Marks : 100

## [Use a separate Answer Book for each Group]

## Group – A

Answer **any six** questions from question nos. **1 to 9**: [6×5]

1. Establish Newton's forward interpolation polynomial formula (without error term) .When it may be used ? [4+1]
2. Deduce numerical differentiation formula from Lagrange's interpolation polynomial. [5]
3. Write the following numbers correct upto 3 significant figures :

   $3.01725, 0.002589.$

   Explain the term 'loss of significant figures' in connection to numerical computation. Give an example of it. [(1+1)+2+1]
4. Find y(4·4) by Euler's modified method taking h = 0·2 from the differential equation :

   $\frac{dy}{dx} = \frac{2-y^2}{5x}$ ; $y = 1$ when $x = 4$, correct upto five places of decimals. [5]
5. Define degree of precision in connection to numerical integration. Using Simpson's $\frac{1}{3}$rd rule evaluate $\int_0^1 \frac{1}{1+x^2}dx$ correct upto 3 decimal places. [1+4]
6. Establish Newton-Raphson method for the real root of an equation f(x) = 0. Give geometrical interpretation of this method. [4+1]
7. Solve the equation $\frac{dy}{dx} = x + y, y(0) = 0$ by 4th order Runge-Kutta method from x = 0 to x = 0·4 with step-length 0·2. Compare the result with exact result (correct upto 4 decimal places). [4+1]
8. a) Discuss the necessity of pivoting in Gauss elimination method with proper example. [4]

   b) State two conditions when power method fails. [1]
9. Find the numerically largest eigenvalue of the matrix [5]

   $A = \begin{pmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{pmatrix}$ correct upto 3 decimal places

Answer **any two** questions from question nos. **10 to 12**: [2×10]

10. a) Show that $\vec{\nabla}(\vec{A}\cdot\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \vec{A}\times(\vec{\nabla}\times\vec{B})$. [5]

    b) Evaluate $\iint_S \vec{A}.\vec{n}dS$ where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2 = 16$ included in the first octant between the planes z = 0 and z = 5. [5]
11. a) Verify Green's theorem in the plane for $\int_C (2x - y^3)dx - xy\,dy$ , where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. [6]

    b) Show that $\iint_S r^5 d\vec{S} = \iiint_V 5r^3\vec{r}dV$, where $r = |\vec{r}|$ and V is the volume bounded by the surface S. [4]

12 a) Find the unit outward drawn normal to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at (3, 1,-4). [4]

b) Find the most general differentiable function f(r) so that $f(r)\vec{r}$ is solenoidal ( where $r = |\vec{r}|$ ). [4]

c) State Stokes' theorem. [2]

## Group – B

Answer **any two** questions from question nos. **13 to 15 :** [2×12]

13. a) If A, B, C; D, E, F be the moments and products of inertia of a rigid body about rectangular axes OX, OY, OZ and if $l_1, m_1, n_1; l_2, m_2, n_2$ be the direction cosines of two lines OL, OK at right angles to each other, then find the moment of inertia of the body about OL and product of inertia about OL and OK. [6]

b) Two equal cylinders of radius *a*, each of mass m, are bound together by an elastic string whose tension is T and roll with their axes horizontal down a rough plane of inclination $\alpha$. Show that their acceleration is $\frac{2}{3} g \sin\alpha \left(1 - \frac{2\mu T}{mg\sin\alpha}\right)$, where $\mu$ is the coefficient of friction between the cylinders. [6]

14. a) A thin rod of length 2*a* revolves with uniform angular velocity $\omega$ about a vertical axis through a small joint at one extremity of the rod, so that it describes a cone of semi-vertical angle $\alpha$.

Show that $\omega^2 = \frac{3g}{4a\cos\alpha}$. Discuss the cases, when $\omega < \sqrt{\frac{3g}{4a}}$ and $\omega > \sqrt{\frac{3g}{4a}}$. [6]

b) A solid homogeneous cone, of height h and vertical angle $2\alpha$, oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(4 + \tan^2\alpha\right)$. [6]

15. a) Prove that the magnitude of the angular momentum of a rigid body moving in two dimensions about a fixed point can be expressed with usual notations, in the form $Mvp + Mk^2\frac{d\theta}{dt}$. [6]

b) A uniform vertical circular plate, of radius *a* , is capable of revolving about a smooth horizontal axis through its centre. A rough perfectly flexible chain, whose mass is equal to that of the plate and whose length is equal to its circumference, hangs over its rim in equilibrium. If one end be slightly displaced, then show that the velocity of the chain when the other end reaches the plate is $\sqrt{\frac{\pi a g}{6}}$. [6]

Answer **any one** question from question nos. **16 to 17 :** [1×6]

16. Two equal uniform rods AB and AC are freely jointed at A. They are placed on a table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to AC. Show that the resulting velocities of the middle points of AB and AC are in the ratio 2:7. [6]

17. A heavy ring, of radius *a*, is moving in its own plane which is vertical. At a certain instant, when its velocity is V horizontally from left to right and the angular velocity is $\frac{V}{2a}$ clockwise, the highest point of the ring is suddenly fixed. Prove that the ring will describe a complete revolution about the point of fixing, if $V^2 > 32\,ag$. [6]

Answer **any two** questions from question nos. **18 to 20 :** [2×7]

18. A heavy particle slides down a rough cycloid of which the coefficient of friction is $\mu$. Its base is horizontal and vertex downwards. Show that if it starts from rest at a point where the tangent makes an angle $\theta$ with the horizontal and comes to rest at the vertex, then $\mu e^{\mu\theta} = \sin\theta - \mu\cos\theta$. [7]

19. A comet is moving in a parabola about the Sun as focus. When at the end of its latus rectum its velocity suddenly becomes altered in the ratio n : 1, where n < 1; show that the comet will describe

an ellipse whose eccentricity is $\sqrt{1-2n^2+2n^4}$ , and whose major axis is $\frac{l}{1-n^2}$, where $2l$ is the latus rectum of the parabolic path. [7]

20. A spherical raindrop, of radius *a*, falls from a height h and acquires moisture from the atmosphere throughout its motion; the radius thereby increases at the rate of *ca*. Show that when it reaches the ground , its radius becomes $ca\sqrt{\frac{2h}{g}}\left(1+\sqrt{1+\frac{g}{2c^2h}}\right)$ . [7]

Answer **<u>any one</u>** question from question nos. **21 to 22** : [1×6]

21. If a planet were suddenly stopped in its orbit, supposed circular, show that it would fall into the Sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. [6]

22. When a periodic comet is at its greatest distance from the Sun, its velocity *v* is increased by a small quantity $\delta v$ . Show that the comet's least distance from the Sun is increased by $4\delta v.\left\{\frac{a^3(1-e)}{\mu(1+e)}\right\}^{\frac{1}{2}}$ , where $2a$ and $e$ are the major axis and eccentricity of the original orbit and $\mu$ is the intensity of the Sun's attraction at unit distance. [6]

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